REVIEW ON ZINGER'S TECHNIQUE FOR VARIANCE ESTIMATION OF SEMI-RANDOM SAMPLING

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ABSTRACT: *This paper provides a discussion of a proposed unbiased variance estimator methodology by Zinger for semi-random sampling. In this investigation, comparisons among some samples estimators are made using a finite population. Adopting some of the surveyed literature, results of variance estimation are illustrated to provide a satisfactory estimator of the variance of the sample mean with proposed additional assumptions. A method is proposed to estimate the mean of a finite population and to estimate the variance of this estimate, using a mixture of two samples drawn from a finite population to get an unbiased estimator for the population variance under study depending on Zinger's methodology.*

Key words: Finite population; Unbiased Estimator; Variance Estimation; Zinger's Technique

1.0. INTRODUCTION:

Searching for desired properties of estimators is extremely important for statisticians since they provide criteria for selecting among alternative different estimators. Discussion on an approach to an unbiased estimator of the mean and variance of a finite population is presented in this article adopting a specific sample technique, which in fact consists of both simple random sampling and systematic sampling.

There are some different approaches to deal with the issue of variance estimation considered in the literature. One among these approaches is to adopt some assumptions about the structure of the population, where the population is assumed randomly, which intensively discussed by *[1]*. The results of these estimates provide generalized unbiased estimates and sometimes conclude to overestimate variances. [1] has discussed the models with linear trend, stratification only and smooth periodic trend. Another approach mentioned by [2],consists of taking more than one sample, a common approach is known as multiple start sampling in which p different random starts k_j, j=1,2,…,P (1≤ k_j ≤,k_p) and obtain p systematic samples each of size $\acute{m}$=m/ p with i=0,1,…,$\acute{m}$ -1. Due to *[3]* and *[4])*,several systematic samples usually give a slightly less precise estimate than a single systematic sample of the same total size , *[5]* article proposed an exponential ratio type estimator to estimate the finite population mean for simple and stratified random sampling in which the estimator found to perform better than the usual mean, ratio, exponential ratio, traditional regression estimators in simple and stratified random sampling. [6] suggested the generalized class of estimators of finite population variance utilizing the known value of parameters related to an auxiliary variable used in simple random sampling without replacement. *[7]* compared estimators for successive difference replication and Ripley's and D'Orazio's variance estimators. The variances obtained concluded that in populations with a near zero spatial autocorrelation, all estimators, performed equally, and produced estimates close to the actual design variance. *[8]* article dealt with the estimation of the finite population mean under probability proportional to size sampling using auxiliary variable and regression type estimators by incorporating the maximum and minimum values of the study variable and the auxiliary variable.The literature related to this topic can be found in many other recent articles(see for instance : [9-[15*]* and*[16]*.

2.1. Approach of Population Parameters Estimation:

In the following sections, let y_{ij} denotes the ***j***-th element of the ***i***-th systematic sample ,so that ***j*** =1,2,3,…,k .The mean of the ***i***-th sample is denoted by $\bar{y}_i$.The systematic sampling estimator of the population mean, denoted by μ :

$$\frac{\sum_{i=1}^{n}(y_i)}{n} \qquad (1)$$

Adopting the variance of the above mean estimator, it requires some assumptions as below:,see *[10]*.

1-The case when the population elements are assumed to be in no specific order with respect to the variance of the estimator of the mean is the same as in the case of simple random sampling *[10]*:

$$S^2(\overline{Y}_{sy}) = \frac{(N-n)}{Nn}S^2 \qquad (2)$$

2-The case where the mean is constant within each stratum of K elements but different from stratum to stratum, the estimated variance of the sample mean will be given by:

$$S^2(\overline{Y}_{sy}) = \frac{\frac{N-n}{Nn}\sum_{i=1}^{n}\left(Y_i - Y_{i+k}\right)^2}{2(n-1)} \qquad (3)$$

3-When the population is either increasing or decreasing linearly in the variable of interest, and when the sample size is large, the appropriate estimator of the variance of this estimator of the mean is:

$$S^2(\overline{Y}_{sy}) = \frac{\frac{N-n}{Nn}\sum_{i=1}^{n}\left(Y_i - 2Y_{i+k} + Y_{i+2k}\right)^2}{6(n-2)} \qquad (4)$$

$for 1 \le I^{/} \le n-2$

According to *[1]* , the variance of the mean of a systematic sample is:

$$V\left(\overline{y}_{sy}\right) = \frac{N-1}{N}.S^2 - \frac{k(n-1)}{N}S_{wsy}^2 \qquad (5)$$

Where :

$$S_{wsy}^2 = \frac{1}{k(n-1)}\sum_{i=1}^{k}\sum_{j=1}^{n}\left(y_{ij} - \overline{Y}_i\right)^2$$

is the variance among units that lie within the same systematic sample. *[1]* also added that:

$$V\left(\overline{y}_{sy}\right) = \frac{S^2}{n}\frac{N-1}{N}[1 + (n-1)\rho_w] \qquad (6)$$

Where :

$\rho$is the correlation coefficient between pairs of units that are in the same systematic sample. It is defined as:

$$\rho_w = \frac{E\left(y_{ij} - \overline{Y}\right)\left(y_{iu} - \overline{Y}\right)}{E\left(y_{ij} - \overline{Y}\right)^2}$$

Which gives:

$$\rho_w = \frac{2}{(n-1)(N-1)S^2}\sum_{i=1}^{k}\sum_{j<u}(y_{ij}-\overline{Y})(y_{iu}-\overline{Y}) \quad (7)$$

And:

$$V\left(\overline{y}_{sy}\right) = S^2_{wst} = \frac{1}{n(k-1)}\sum_{j=1}^{n}\sum_{i=1}^{k}\left(y_{ij}-\overline{Y}_j\right)^2,$$

Where

$$\rho_{wst} = E(y_{ij}-\overline{Y}_j)(y_{iu}-\overline{Y}_u)$$

$$\rho_{wst} = \frac{2}{n(n-1)(k-1)}\sum_{i=1}^{k}\sum j<u\ \frac{\left(y_{ij}-\overline{Y}\right)\left(y_{iu}-\overline{Y}_u\right)}{S^2_{wst}}$$

The above quantity is the correlation between the deviations from the stratum means of pairs of items that are in the same systematic sample. Therefore, a systematic sample has the same precision as the corresponding stratified sample, with one unit per stratum, if$\rho_w = 0$. The variance of the mean from stratified sample is :

$$V(\overline{y}_{st}) = \left(\frac{N-n}{N}\right)\frac{S^2_{wst}}{n} \quad (8)$$

and the variance $V(\overline{y}_{st})$is found directly from the systematic sample totals as:

$$V(\overline{y}_{st}) = V_{sy} = \frac{1}{k}\sum_{i=1}^{k}(\overline{y}_i-\overline{Y})^2$$

$$= \frac{1}{n^2k}\sum_{i=1}^{k}(n\overline{y}_i-n\overline{Y})^2 \quad (9)$$

Caution must be taken when performing variance estimation in systematic sampling which needs gathering much prior information as possible about the nature and ordering of the population as well as using auxiliary variable to construct a simple model for the population, and due to multipurpose of most of surveys , it may be important to use different variance estimators for different characteristics. This suggests that one should know the population well before choosing a variance estimator, which is exactly the advice most authors have suggested before using systematic sampling *[3]*.

2.2.Proposed Estimator of the Variance:

Consider a finite population consists of units numbered from I to $N = m.k$,where **m** and **k** are positive integers. Let the values of the units be$x_i, i = 1,2,3,\ldots.N,$, with mean T. Let a random integer $k_0$be taken such that $1 \le k_0 \le k$.The sample $xk_0 + i_k, i = 0,1,\ldots,m-1$is called a systematic sample of size **m**. There are **K** possible and equally probable samples. Let $\overline{X}$be the sample mean of a systematic sample of size **m.** According to *Cochran [1]*,we have :

$E(\overline{X}_s) = \overline{T}$ and

$$V(\overline{X}_s) = \left(\frac{S^2}{m}\right)(N-1)/N)((1+(m-1)\rho_w) \quad (10)$$

Where

$S^2 = \frac{\sum_{i=1}^{N}(x_i-\overline{T})^2}{(N-1)}$ and ρ_w is the correlation coefficient between pairs of units that are in the same systematic sample, where:

$$\rho_w = \frac{2}{(m-1)(N-1)S^2}\sum_{k_0=1}^{k}\sum_{j<u}(X_{k_0+jk}-\overline{T})(X_{k_0+uk}-\overline{T}) \quad (11)$$

Then it can be reached that the within-sample variance leads on the average to an underestimate of $V(\overline{T}_s)$ if $\rho_w > 0$or to an overestimate if$\rho_w < 0$.For semi random sampling, first a systematic sample of size **m** is drawn .Then a simple random sample of size **n** is drawn from the remaining **N- m** units, if $(\overline{X}_s)$ and $(\overline{X}_r)$ denote the means of' these two samples respectively. This technique will be referred to as semi random sampling (SRS).We shall take the following weighted means of $(\overline{X}_s)$ and $(\overline{X}_r)$as an estimator of $\overline{T}$:Using the conditional expectation $E_r$over random sampling with a fixed systematic sample and then **k**, over systematic sampling. Following *Zinger[2]*,we have:

$$E\left(\overline{X}(\beta)\right) = E_sE_r[\alpha\overline{X}_s + \beta\overline{X}_r]$$

$$= E_s[\alpha\overline{X}_s + \beta(T - m\overline{X}_s)/(N-m)]$$

Where $T = N\overline{T}$.

To compute

$$V(\overline{X})\beta = E_sE_r\left\{[\alpha\overline{X}_s + \beta\overline{X}_r]^2\right\} - \overline{T}^2$$

We have

$$E_sE_r\left(\overline{X}_s^2\right) = V(\overline{X}_s) + \overline{T}^2 \quad (12)$$

$$E_sE_r(\overline{X}_s\overline{X}_r) = \frac{T\overline{T}}{N-m} - \frac{m}{N-m}E_sE_r(\overline{X}_s)^2 \quad (13)$$

Combining these results and get

$$E_sE_r\left(\overline{X}_r^2\right) = \frac{(n-1)T^2}{n(N-m)(N-m-1)} - \frac{2(n-1)T\overline{T}}{n(k-1)(N-m-1)} + \frac{m(n-1)}{n(k-1)(N-m-1)}E_sE_r\left(\overline{X}_s^2\right) + \frac{(N-m-n)(k-1)}{nk(N-m)(N-m-1)}$$

$$(N\overline{T}^2 + (N-1)S^2 \quad (14)$$

Combining and using (3 12), (3.13) and (3.14) we can write $V\left(\overline{X}(\beta)\right)$ as

$$V\left(\overline{X}(\beta)\right) = \alpha_1(\beta)T^2 + \alpha_2(\beta)S^2 + \alpha_3(\beta)V(\overline{X}_S), \quad (15)$$

Where

$$\alpha_1(\beta) = 0$$

$$\alpha_2(\beta) = \frac{\beta^2(N-1)(N-m-n)}{nN(N-m-1)},$$

$$\alpha_3(\beta) = (1 - \frac{k\beta}{(k-1)^2} - \frac{(N-m-n)\beta^2}{(n(k-1)^2(N-m-1))} \quad (16)$$

To compute the $\boldsymbol{V\left(\overline{X}(\beta)\right)}$ let us consider the sample sum of squares.

$\boldsymbol{\phi(\beta) = \sum_{i=1}^{m+n}\left(x_i - \overline{X}(\beta)\right)^2}$**,**and compute

$$E(\phi(\beta)) = E_sE_r\left[\sum_{i=1}^{m+n}(x_i - \alpha\overline{X}_s - \beta\overline{X}_r)^2\right]$$

Using

$$E_sE_r\left(\sum_{i=1}^{m} x_i^2\right) = E_s\left(\sum_{i=1}^{m} x_i^2\right) = \sum_{i=1}^{m} x_i^2/k$$

$$E_sE_r\left(\sum_{i=1}^{n} x_i^2\right) = E_s n\left(\sum_{i=1}^{N-m} x_i^2\right)/(N-m) = n\sum_{i=1}^{n} x_i^2/N$$

We can then conclude to :

$$C_2(\beta) = \frac{N-1}{N}\left[m+n-2\beta\frac{N-m-m}{N-m-1}+(m+n)\frac{N-m-n}{n(N-m-1)}\beta^2\right]$$

$$C_3(\beta) = \frac{m+n+kn-N}{k-1} - \frac{2\beta}{k-1}\left[n+nk+\frac{m(n-1)}{N-m-1}\right] + \frac{(m+n)}{(k-1)^2}\left[k^2-\frac{N-m-n}{n(N-m-1)}\right]\beta^2 \qquad (18)$$

Now we can adopt the problem of estimating $V\left(\overline{X}(\beta)\right)$ to find the values of **A** and β such that:

$E(A\phi(\beta) = V\left(\overline{X}(\beta)\right)$, Where **A** is nonzero constant. We can obtain β as the solution of:

$\alpha_2(\beta)C_3(\beta) - \alpha_3(\beta)C_2(\beta) = 0$ (19)

If **m** and **n** are fixed and **k** increases, we can find that:

$V\left(\overline{X}(\beta)\right) \to V\left(\overline{X}(1)\right) = S^2/n$

and the efficiency, eff, of $\overline{X}(\beta)$defined as $eff = S^2/\left[(m+n)V(\overline{X}(\beta))\right]$

tends to $n/(m+n)$. But this approach seems unsatisfactory. Another way of estimating V($\overline{X}$ (β) is to consider any two values ofβ, sayβ_1, andβ_2, such that $C_2(\beta_1)C_3(\beta_2) \neq C_2(\beta_2)C_3(\beta_1)$ and use (17) by replacing all expected values by the corresponding sample values. Solving

$$\left.\begin{matrix}\phi(\beta_1) = C_2(\beta_1)S^2 + C_3(\beta_1)v \\ and \\ \phi(\beta_2) = C_2(\beta_2)S^2 + C_3(\beta_2)v\end{matrix}\right] \qquad (20)$$

We obtain the unbiased estimators.

$$S^2 = \frac{N}{(N-1)}\left\{\frac{\alpha_2\phi(0) - n\alpha_1\left(\overline{X}_s - \overline{X}_r\right)^2}{(m+n)\alpha_2 - \alpha_1(N-m-n)/(N-m-1)}\right\}$$

Of S^2 and

$$V = (k-1)\left[\frac{(N-m-n)\phi(0)/(N-m-1) - n(m+n)\left(\overline{X}_s - \overline{X}_r\right)}{(N-m-n)\alpha_1/(N-m-1) - (m+n)\alpha_2}\right]$$

Where :$\alpha_1 = m+n+kn-N$,and $\alpha_2 = nk+n+[m(n-1)/(N-m-1)]$, we can obtain these results using any $\beta_1$and β_2 as long as:

a. $C_2(\beta_1)C_3(\beta_2) \neq C_2(\beta_2)C_3(\beta_1) and m \neq nor$ $\beta_1 + \beta_2 \neq 1 if m = n$

It can be seen that$S^2 \geq 0$, which means that this procedure provides an unbiased and positive estimator ofS^2. On the other hand, $\boldsymbol{v}$ is an unbiased estimator of $V\left(\overline{X}_s\right)$,but is not always positive as *Zinger* (1980).

Using (15) and (16), we obtain the unbiased estimator.

$v\left(\overline{X}(\beta)\right) of V\left(\overline{X}(\beta)\right)$,We now have, for all β'S

$v\left(\overline{X}(\beta)\right) = \alpha_2(\beta)S^2 + \alpha_3(\beta)v$ (21)

And $Ev\left(\overline{X}(\beta)\right) = V\left(\overline{X}(\beta)\right)$.

$E(\phi(\beta) = C_1(\beta)T^2 + C_2(\beta)S^2 + C_3(\beta)V\left(\overline{X}_s\right),$ (17)

Where :$C_1(\beta) \equiv 0$

There are choices for βhave been considered.

1. The value of β, which immunized :$\alpha_2(\beta)S^2 + \alpha_3(\beta)v$. This produces negative estimates of $v\left(\overline{X}(\beta)\right)$for some samples and is not retained.

The value$\beta = n/(m+n)$, corresponding to a natural weight of $\overline{X}_s$and$\overline{X}_r$,fo1lowing *Zinger* (1980) , we can say that if:

$\rho_w < -1/(N-1), then$

$V\left(\overline{X}(n/m+n))\right) < \frac{N-m-n}{N(m-n)}S^2$ (22)

The value which is like the relationship between the variances of the means for SRS and systematic sampling. Unfortunately, some $v\left(\overline{X}(n/m+n))\right)$ are found negative and this choice is therefore not prefered.The value$\beta =$ (½), corresponding to an un-weighted average of $\overline{X_s}$, and $\overline{X_r}$ produces positive results for $v\left(\overline{X}\right)$ (½) and is recommended for this reason. It can be shown also, by minimizing$V\overline{(X}(\beta)$, that if **K** is large, the value $\beta = {}^1/_2$is optimal for $n = m/(1+(m-1)\rho_w$ which gives the best choice for **n** if ρ_w is known.

2. The value which is like the relationship between the variances of the means for SRS and systematic sampling. Unfortunately, some $v\left(\overline{X}(n/m+n))\right)$ are found negative and this choice is therefore not preferred value$\beta =$ (½), corresponding to an un-weighted average of $\overline{X_s}$, and $\overline{X_r}$ produces positive results for $v\left(\overline{X}\right)$ (½) and is recommended for this reason. It can be shown also, by minimizing$V\overline{(X}(\beta)$, that if **K** is large, the value $\beta = {}^1/_2$is optimal for $n = m/(1+(m-1)\rho_w$ which gives the best choice for **n** if ρ_w is known.

3..Results:

In this section, an illustration of results concerning the estimation of the underlying technique is demonstrated. We consider data show the main annual food crops production of the Sudan. These crops are mainly sorghum, wheat, and millet. Each of these is considered as a separate population ,denoted by population 1 population 2,and population 3 for these products respectively, where the total population size N =25 for each population.

Table(1):Analysis of Variance

	d.f	s.s	m s
Between rows(strata)	4	**6634830**	**1658707.5**
Within strata	**20**	**16134290**	**806714.5 =** s^2_{wst}
Totals	**24**	**22769120**	**2465422 =** s^2

Table(2) :Analysis of Variance

	d.f	s.s	m s
Between rows(strata)	4	**617361.2**	**154340.3**
Within strata	**20**	**233282.8**	**11664.1 =** s^2_{wst}
Totals	**24**	**850644.5**	**166004.4 =** s^2

Table(3) :Analysis of Variance

	d.f	s.s	m s
Between rows(strata)	4	**1443246.8**	**36011.7**
Within strata	**20**	**683703.2**	**34185.2=**s^2_{wst}
Totals	**24**	**2126950**	**394996.9 =** s^2

For random and stratified sampling ,analysis of variance of the population into" between rows " and "within rows " is presented in table (1), table(2) and table(3),.while table(4), is specified for the mean and variance of specific sampling methods for the three populations, while the distribution of the variance is presented in table(5).

The purpose for the analysis of variance is to test for significant differences between means. The within group variability is usually referred to as error variance. If the strata are homogeneous, the variability within-groups is expected to be lower than the variability for the population as a whole.It is seen that $\boldsymbol{V(\bar{y}_{st})}$ is always less than $\boldsymbol{V_{ran}}$ and $\boldsymbol{V(\bar{y}_{st})}$ in population 1,2 and 3.But $\boldsymbol{V(\bar{y}_{sy})}$ is less than $\boldsymbol{V_{ran}}$ in two cases out of three as can be noted in population 2.This may be conducted to the distribution of the data as shown in table(4).

Table(4):Summary of Results of Mean and Variance of Specific Sampling Methods.

	Pop 1	Pop 2	Pop 3
Population Mean N=25	**2319.9**	**285**	**368.8**
$\boldsymbol{s^2}$	**2465422**	**16664.4**	**394996.9**
$\boldsymbol{V_{ran}}$	**473361**	**26560.7**	**63199.5**
$\boldsymbol{V(\bar{y}_{st})}$	**154889.2**	**1866.3**	**5469.3**
$\boldsymbol{V(\bar{y}_{sy})}$	**4968739.3**	**705.42**	**874.7**

To illustrate the variance estimation for our underlying method of sampling, let us consider the populations 1,2, and 3 which are denoted by pop1,pop2, and pop3 respectively, with N =25,k=5.The results of variance estimation are summarized in tables(5),(6) and (7).

In order to obtain the distributions of S^2 ,$V(\bar{y}\left(\frac{n}{(m+n)}\right))$ and $V\left(\bar{y}(1/2)\right)$,all possible samples were considered for **n= 1,2,3** .Table (8) gives some results for the distribution of S^2 . Table (6) gives some results for the distribution of $V(\bar{y}\left(\frac{n}{(5+n)}\right))$,as table (7) gives some results for the distribution of $V\left(\bar{y}(1/2)\right)$,and the distribution of $V(\bar{y})$ using simple random sampling of size **n .**

It should be noted that the results in table (6) are the obtained unbiased estimators of $\boldsymbol{S^2}$ of $\boldsymbol{S^2}$ .It is seen that $\boldsymbol{S^2} > 0$ or in fact $S^2 \geq 0$ which means that this procedure provides an unbiased and positive estimator, of $\boldsymbol{S^2}$**.**On the other hand ,**v** is an unbiased estimator of $\boldsymbol{V(\bar{y}_{sy})}$**,**but is not always positive .For instance, if n=1 the values of **v** are -13.24,-15.81 and 4.6 for pop1,pop2 and pop3 respectively.

Table(5):Distribution of $\boldsymbol{S^2}$

	n	Min	Max	S.e
SRS Pop 1 m=5	**1**	**16344.5**	**2057280.5**	**1538**
	2	**8172.3**	**982172.8**	**1065**
	3	**5448.2**	**618718.2**	**850**
	4	**4086.1**	**436982.1**	**660**
	5	**3286.9**	**327941.3**	**628**
SRS Pop 2 m=5	**1**	**2.3204**	**159322.5**	**391**
	2	**1.1602**	**75468.6**	**276**
	3	**0.7735**	**47517.3**	**221**
	4	**0.5800**	**33541.6**	**187**
	5	**0.4640**	**25156.3**	**163**
SRS Pop 3 m=5	**1**	**2.8773**	**399145.2**	**616**
	2	**1.4382**	**179595.2**	**426**
	3	**0.9591**	**113078.5**	**340**
	4	**0.7193**	**79820.1**	**288**
	5	**0.5755**	**59865.2**	**251**

Note: The skewness of :

Pop1= 0.7326 Pop2 = 0.8496 Pop3 = 0.2331

It worth remarking that the value $\beta = ½$ corresponding to an unweighted average of $\overline{\boldsymbol{y}}_{sy}$ and $\overline{\boldsymbol{y}}_{ran}$ producing a positive result of $V(\bar{y}\,(^1/_2))$ as can be noted in table (7).Therefore, $\bar{y}\,(^1/_2)$ is recommended for this reason.

Table(6):Distribution of $V(\bar{y}\left(\frac{n}{5+n}\right))$

	n	Min	Max	S.e
SRS Pop 1 m=5	**1**	**14200877**	**23668018**	**1538.4**
	2	**16567714**	**982172.8**	**1064.9**
	3	**9047336**	**618718.2**	**850**
SRS Pop 2 m=5	**1**	**2.3204**	**159322.5**	**391**
	2	**1.1602**	**75468.6**	**276**
	3	**0.7735**	**47517.3**	**221**
SRS Pop 3 m=5	**1**	**2.8773**	**379145.2**	**616**
	2	**1.4382**	**179595.2**	**426**
	3	**0.9591**	**113078.5**	**340**

Now we compare Multiple- Start Systematic Sampling (MSSS) with Semi Random Sampling(SRS) bases on **m+n** observations. In this case ,several conditions should be fulfilled .Given a finite population of units y_i numbered from **1** to **N=mk** ,we draw without replacement **p** systematic samples ,each of size **m** so that **pm = m+n** (total sample size). **mk =N**, where **m** and **k** should be integers. We denote by $V(\bar{y}_{MSSS})$ the variance of the mean, of the **p** systematic samples, each of size **m** ,Then a comparison of MSSS and SRS will be made, Results are shown in table (8) using the previous three populations, where the fourth population, is an artificial borrowed population of the 24 values :13,7,6,7,19,8,8,2,5,20,18,4,0,12,9,16,10,17,15,1,1,6,20,16. was sampled for comparison. Some remarks can be drawn from the results of table(11):

Table(7) :Estimated Value for $V(\bar{x}\,(^1/_2))$

Sampling Method	n	$V(\bar{x}\,(\beta)) = V(\bar{x}\,(^1/_2))$
Semi Random Sampling(SRS) Pop 1 m=5	**1**	**516375.19**
	2	**244653.17**
	3	**154084.04**
SRS Pop 2 m=5	**1**	**39830.36**
	2	**18867.15**
	3	**11879.23**
SRS Pop 3 m=5	**1**	**94785.99**
	2	**44898.6o**
	3	**28269.50**
Simple random sample	**5**	**394467.52**
	6	**21027.22**
	7	**40628.25**

Note: For simple random sample**,** $= (\frac{N-n}{nN})S^2$ **.**

Multiple-start systematic sampling(MSSS),in general, can lead to lower values of the estimated variance as can be seen in table(8),for population 4,and semi-random sampling(SRS) can lead to higher values of the estimated variance. In addition, there are seen negative values of the estimated variances for both MSSS and SRS as in both pop1 and pop2.This may not be due to the conduct of the data but can be a disadvantage of the estimator itself. In addition to that,there is an irregular variation of ρ_w ,as a function of **m** , and no simple relation is evident even if m $m' =$ **m** ,the conclusion that SRS is better if $\rho_{w>0}$ and MSSS is better if $\rho_{w<0}$ does not hold in general.The advantage of SRS is that it provides an estimator of both S^2 and $V(\bar{y}\,(^1/_2))$.

Table(8):Comparison of Estimated Variances Distributions

Pop No.	Sampling Method	$\acute{m}$ m	p n	$\acute{p}_w$ p_w	Min	Max	S.e
1	**MSSS**	**4**	**2**	**-0.90**	**8626.28**	**984486.2**	**1064.94**
	SRS	**4**	**4**	**-0.30**	**310546.2**	**806952.4**	**720**
2	**MSSS**	**4**	**2**	**-0.97**	**1.101**	**75678.17**	**276.34**
	SRS	**4**	**4**	**-0.33**	**705.4**	**83973**	**186.8**
3	**MSSS**	**4**	**2**	**-0.99**	**1.367**	**180094**	**408.86**
	SRS	**4**	**4**	**-0.33**	**874.7**	**20077.9**	**288.7**
1	**MSSS**	**5**	**4**	**-0.30**	**- 61291.9***	**436982.72**	**719.5**
	SRS	**5**	**3**	**-0.45**	**5448.2**	**618718.46**	**850.4**
2	**MSSS**	**5**	**4**	**-0.32**	**0.580**	**33541.65**	**186.7**
	SRS	**5**	**3**	**-0.49**	**-12.375***	**47517.29**	**220.7**
3	**MSSS**	**5**	**4**	**-0.33**	**0.719**	**79820,14**	**288**
	SRS	**5**	**3**	**-0.49**	**.959**	**113078.47**	**340.4**

Table(8):Comparison of Estimated Variances Distributions (continued)

4	**MSSS**	**2**	**2**	**0.10**	**0.00**	**63.80**	**12.42**
	SRS	**2**	**2**	**0.10**	**0.05**	**64.84**	**10.66**
	SRS	**3**	**1**	**-0.09**	**0.41**	**48.39**	**10.66**
4	**MSSS**	**2**	**3**	**0.10**	**0.08**	**23.52**	**5.03**
	MSSS	**3**	**2**	**-0.09**	**0.08**	**21.33**	**5.20**
	SRS	**2**	**4**	**0.10**	**0.02**	**57.87**	**7.89**
	SRS	**3**	**3**	**0.09**	**0.13**	**37.76**	**4.89**
	SRS	**4**	**2**	**0.21**	**0.37**	**37.33**	**6.59**
4	**MSSS**	**4**	**3**	**0.21**	**0.46**	**7.19**	**1.97**
	MSSS	**6**	**2**	**0.50**	**0.01**	**3.13**	**1.19**
	SRS	**8**	**4**	**-0.10**	**0.87**	**5.44**	**0.53**

4. DISCUSSION

General results show that the systematic sample mean can be more precise estimate than the mean of a random sample of equal size. However, the sampling variance of the mean of a systematic sample from a list can be expected to be less than that of the mean of a random sample, if there is a consistent trend throughout the list or certain grouping criterion in the values of the population ,although this sampling variance can tend to be biased when estimated from a random systematic sample. It is ,then , obvious that our SRS from which we are estimating variance is trying to get benefit from the advantages of both systematic and simple random sampling .In this technique, the stratification of a population must be judged by the investigator's knowledge of the population and surveys carried on similar populations. Semi random sampling technique which used here is expected to lessen the danger in systematic sampling when the characteristics being studied may have a certain pattern or periodicity or trend in the list. Comparing multiple-start systematic sampling with semi random sampling - put into account the required conditions - it is found that multiple-start systematic sampling can lead to lower values of the estimated variances .However, providing more than estimator is an advantage of semi random sampling technique over multiple start systematic sampling adopted by zinger.

5. CONCLUSION

The paper has investigated the problem of an unbiased variance estimator for semi random sampling technique and the related sampling methods .The underlying proposed method used a systematic sample and a simple random sample drawn from the remaining population to obtain an unbiased estimator of the population variance. Results of estimation for semi random sampling technique does not provide a satisfactory estimator of the variance of the sample mean unless additional assumptions are made. Hence, when considering the problem of estimation for this method, a great caution must be taken about the structure and ordering of the population under study and several variance estimators can be used to choose the best and efficient estimator. In addition to that, semi random sampling is advised to use only if there are several samples to be taken and a great deal of care must be taken in the analysis of data.

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